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# Influence of the Phase-Locked Loop on the Design of Microgrids Formed by Diesel Generators and Grid-Forming Converters

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Abstract-In recent years, microgrids (MGs) with renewable energy sources, diesel gen-sets and droop-controlled converters have been increasingly used to guarantee the continuity of power supply in remote areas. Renewable energy sources have been typically connected to MGs by using an electronic converter that features the two controllers: a current-control loop and a phaselocked loop (PLL). Stability issues related to the PLLs application in electrical grids have already been addressed in the literature, however, dynamic interactions in MGs caused by PLLs have not been sufficiently explored. In this paper, a MG that includes a grid-feeding voltage source converter and a grid-forming device (diesel gen-set or converter) is studied. All network elements are modelled analytically and the eigenvalue and participation-factor analyses are used to analyse the interactions between the devices. It is demonstrated that MGs formed by diesel gen-sets have reduced stability limits. Also, it is shown that stability margins of MGs formed by droop-controlled converters can be improved by changing the control parameters (e.g. PLL and internal controllers bandwidths). The main findings and conclusions are summarised and presented as a practical MG design guide. Theoretical results are validated in a lab environment comprising two 15 kW converters and one 75 kW grid emulator.

*Keywords*—Microgrid, Phase-Locked Loop, Current Control, Diesel Generator, Droop Control, Small-Signal Analysis.

#### NOMENCLATURE

#### Variables State variable of the AVR $\alpha$ β State variable of the speed governor δ Relative angle of the reference frame Angle generated by the PLL $\delta_{PLL}$ State variable of the PLL $\gamma^{dq}$ State variable of current controller Droop nominal angular frequency $\omega_{nd}$ MG nominal frequency $\omega_n$ Angular frequency generated by the PLL $\omega_{PLL}$ $\omega_{sq}$ in pu $\omega_r$

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- $\omega_{sg}$  Angular frequency of the generator rotor
- $\phi^{dq}$  State variable of voltage controller
- $\psi$  Stator flux linkage
- $\psi_f, \psi_k$  Rotor flux linkages
- P, Q Filtered active and reactive power
- f Injected fuel
- $i_f, i_k$  Rotor currents
- $i_i, v_c, i_o$  Internal current, capacitor voltage and external current of the LCL filter
- $i_l$  Current absorbed by the load
- $i_{sg}$  Stator current
- $t_e$  Electric torque
- $t_m$  Mechanical torque
- $v_f$  Generator field voltage
- $v_i$  Voltage command generated for the modulation stage
- $v_{nd}$  Droop nominal voltage
- $v_{pcc}$  Voltage at the connection point
- $v_r$  Regulated voltage
- $v_{sq}$  Stator voltage

# Parameters

- $l_d, l_q$  Self-inductances of stator circuits
- $l_{ad}$ ,  $l_{aq}$  Stator mutual inductances

 $l_{afd}$ ,  $l_{akd}$ ,  $l_{akq}$  Mutual inductance stator-rotor

- $l_{ff}, l_{kkd}, l_{kkq}$  Self-inductances of rotor circuits
- *l*<sub>fkd</sub> Mutual inductance field-damping windings
- $r_a$  Armature resistance per phase in pu

 $r_f, r_{kd}, r_{kq}$  Rotor circuit resistances in pu

#### Constants

- $\sigma$  Degree of increased retarding of the engine
- $\tau_m$  Constant term of the engine time constant
- *H* Per unit inertia constant of the shaft
- $K_E$  Self-excitation constant of the excitation system
- $S_E$  Parameter that models the saturation function of the excitation system
- $T_E$  Time constant of the excitation system

# **Control Parameters**

- $k_{iC}$  Integral constant of the controller C
- $k_{pC}$  Proportional constant of the controller C

#### **Base values**

 $U_B$ ,  $I_B$ ,  $\omega_B$  Base of voltage, current and angular frequency General definition of variables

- $x^{DQ}$  Variable x is represented in the D-Q axis
- $x^{dq}$  Variable x is represented in the d-q axis
- $X_i^j$  Value of variable  $x_i^j$  at the operating point

 $x_i^{*j}$  Set-point of the variable  $x_i^j$ 

#### I. INTRODUCTION

The concept of microgrid (MG) was originally introduced as an alternative solution for electricity supply in remote residential and industrial areas, and physical islands [2,3]. These small-scale grids have been custom made according to their specific application requirements and the availability of primary resources. In a MG additional energy storage and generation devices (such as batteries and diesel gen-sets) may be required to guarantee the continuity of power supply [4, 5]. In islanded MGs, a simple control solution is to use a gridforming (GFo) device to energise the network and set the MG voltage and frequency and to allow the connection of other devices operating in grid-following mode, also known as gridfeeding (GFe) mode [6,7]. In the past, diesel gen-sets have been exclusively used as GFo devices, however, in recent years there have been more examples of droop-controlled power converters taking over this role. The most common converter topology for GFe devices is a voltage source converter (VSC) that comprises a current controller and a phase-locked loop (PLL).

A control system based on a PLL performs correctly in stiff grids and when the network dynamics is relatively slow [7]. However, it has been demonstrated that the PLL design of a GFe-VSC (the PLL, from now on) has a strong influence on the stability of grid-tied converters and the network as the whole. The instability of a grid-connected converter may result from a negative resistance found in its dq impedance model that is related to the PLL bandwidth and the power levels injected to the grid [8]. Indeed, it has been found that the PLL bandwidth limits the maximum power transfer in highvoltage dc interconnections [9]. The interaction between the PLL and dc-voltage controller can also have a negative effect on the stability of grid-connected converters that operate as rectifiers [10]. Some recommendations on the design of current controllers, PLLs and dc-voltage controllers for grid-connected converters were presented in [10]. Since MGs have lower inertia when compared to conventional grids, it is important to verify if PLL introduces the same stability restrictions, as well as to analyse other aspects that may compromise the converter and grid stability.

The effect of PLL-SG interactions on the stability of MGs has been previously studied in the literature [1, 11–14]. Katiraei *et al.* [11, 12] corroborate that the power injected by a GFe converter and the design of current controller and PLL, also in MGs, have an important influence on the stability. By using eigenvalue analysis, it was found that the AVR, the excitation and the generator inertia affect the stability of MGs with SGs and current controlled converters [14]. The interaction between the two aspects (loading conditions and PLL design) was studied in [1]. It was shown that loading conditions and PLL design affect the same eigenvalues and therefore loading conditions should be considered in the design of the PLL. The stability of a MG consisting of droop controlled converters and a controllable load (interfaced by a series-shunt converter that uses PLL) was studied in [15]. In that work, it was found

that the PLL has a little contribution on the oscillatory modes that mainly depend on the droop parameters. Also, interactions between the dc-voltage controller and other controllers of the VSC were studied in [16, 17]. It was found that these interactions are more pronounced in weak grids. In particular, it was found that the stability margins are reduced when the dc-voltage controller and PLL have similar bandwidths.

PLL implementations in conventional grids have been extensively studied [8–10]. The interactions of PLL with the current controller and the dc-voltage controller have also been studied in conventional grids and MGs. However, interactions between different control loops of all the connected devices such as AVR, excitation, droops, and voltage and current controllers of GFo VSCs have not been studied in detail. In addition, for the case of MGs, there are no clear guidelines for the design of the controllers, such as those already existing for VSCs connected to conventional grids. Furthermore, the models of SGs used to analyse stability commonly neglect the damping windings [14, 18], although several publications have taken them into account [11, 12]. Finally, small-signal models of such hybrid and diverse MGs have been rarely validated experimentally [12].

Therefore, according to the author's best knowledge, the dynamic interactions caused by PLLs in MGs (formed by either diesel gen-sets or droop-controlled converters) have not been still sufficiently explored. Also, no general design rules have been proposed to assist the designer in choosing the control parameters in MGs formed of GFo and GFe devices.

This paper focuses on the interaction between grid-feeding and grid-forming devices in MGs. The small-signal models of MGs formed by either diesel generator or droop controller VSC are developed and used to study the influence of the GFe-PLL on the stability of the MG. The main contributions of the paper are:

- Study of dynamic interactions in islanded microgrids between typical grid-forming elements and grid-feeding converters.
- The impact of grid-feeding converters and their PLL on the stability of MG for different network scenarios is studied by using participation factors and eigenvalue analysis.
- Detailed design guidelines are proposed to help MG designer choose and tune control parameters and guarantee the MG stability.
- Comparison between the two most common GFo devices, namely, a diesel gen-set and a droop-controlled VSC with respect to MG stability.

Other contributions of the paper are:

- Development of comprehensive small signal models of microgrids formed by, in one case, a diesel generator and, in the other, a droop controlled VSC.
- Experimental validation of small-signal models and stability boundaries using an experimental platform based on two 15 kW VSCs, one 75 kW grid emulator and 30kW loadbank.

This paper is organised as follows. The MGs topology is described in Section II. Section III explains the small-signal



Fig. 1. Electrical and control diagram of a diesel gen-set and a GFe-VSC connected to a common bus (PCC) feeding a load [1] (Fig.1.).



Fig. 2. Electrical and control diagram of the GFo-VS.

model of each device and of the complete MGs. Section IV studies the interactions between the PLL and the rest of elements using eigenvalues and participation factors. The design guide is developed in Section V, while experimental results are presented and discussed in Section VI. In Section VII results are discussed, including the effect of additional control loops and advanced synchronisation algorithms. Conclusions are drawn in Section VIII.

## II. APPLICATION OVERVIEW

# A. System Description

Fig. 1 shows the MG studied in this paper. The grid can be formed either by a diesel generator (Fig. 1, in the right) or a droop-based converter (Fig. 2). The MG also includes a GFe-VSC and a load. All the devices are connected to the same point of common coupling (PCC). This topology may be used for energy supply of remote residential or industrial areas [2, 3].

The diesel generator includes a diesel engine mechanically coupled to a SG that has its excitation system formed of an alternator and a diode rectifier. The PCC voltage and frequency are controlled by the AVR and the speed governor of the SG, respectively. The AVR generates the set-point for the excitation system  $(v_r)$  while the speed governor generates the set-point for the fuel injection (f).

In order for this study to be applicable to larger MGs and especially MG power parks (GFo converters all connected to the same busbar), we considered GFo-VSC with *LCL* filters. These filters are commonly used to interface GFo-VSC in MGs [19] or to provide the connection to the PCC. Current and voltage loops are used to control the ac capacitor voltage. Also, voltage and frequency droop controllers are used to set the MG voltage and frequency.

The GFe-VSC is connected to the PCC via an LCL filter. The active and reactive power injections are controlled

indirectly by using a current controller implemented in *dq*. The SRF is obtained by using a PLL [7]. *B. Reference Frames* 

The small-signal models have been developed following the methodology presented in [19]. State-space equations are represented in a common reference frame (DQ) synchronised with the output voltage of the GFo device (SG or droopcontrolled VSC). An additional reference frame (dq) generated by the PLL is used to model the GFe-VSC. Variables are transformed between the reference frames by using [19]:

$$\begin{bmatrix} f^D\\ f^Q \end{bmatrix} = \begin{bmatrix} \cos\delta & -\sin\delta\\ \sin\delta & \cos\delta \end{bmatrix} \begin{bmatrix} f^d\\ f^q \end{bmatrix},$$
 (1)

where  $\delta$  is the angle of the dq reference frame with respect to the DQ reference frame.

#### III. SMALL-SIGNAL MODELLING OF MG DEVICES

The small-signal models of the diesel generator, the GFo-VSC and the GFe-VSC are introduced in this section, while their detailed models are provided in the Appendix. The notation is defined in the Nomenclature.

# A. Small-Signal Model of a Diesel Generator

The gen-set model consists of the equivalent electrical circuit of the SG, the excitation circuit, the prime mover (diesel engine), the controllers mentioned above (voltage and frequency) and a shaft model [12, 18, 20–22]. The SG equations are presented in pu and are in the (DQ) reference frame [18].

1) Electromechanical Model: The SG is modelled by a set of differential equations with flux linkages as state variables and a set of algebraic equations that links the currents and the flux linkages [18]. A three-wire system has been considered, so o-axis equations are omitted [18]. A laminated salientpole machine has been considered. Therefore, the small-signal model has five states,  $x_{\psi} = [\psi_D, \psi_f, \psi_k^D, \psi_Q, \psi_k^Q]^T$  (see Nomenclature for details):

$$\Delta \dot{x}_{\psi} = A_{EM} \Delta x_{\psi} + B_{EM1} \left[ \Delta v_f \right] + B_{EM2} \left[ \Delta v_{sg}^{DQ} \right] + B_{EM3} \left[ \Delta \omega_r \right]$$
(2)

where  $A_{EM}$ ,  $B_{EM1}$ ,  $B_{EM2}$  and  $B_{EM3}$  are calculated analytically according to (43) and (48) (in the Appendix). The output variables are the electric torque, the stator currents and the field current. Currents can be calculated from flux linkages using (38)–(42), while the electric torque is [18]:

$$t_e = i_{sg}^Q \psi^D - i_{sg}^D \psi^Q. \tag{3}$$

The linearised expressions for the electric torque and the stator and field currents are:

$$\Delta t_e] = C_{EM1} \Delta x_{\psi}, \qquad (4)$$

$$\begin{bmatrix} \Delta i_{sg}^{D} \\ \Delta i_{sg}^{Q} \end{bmatrix} = C_{EM2} \Delta x_{\psi}, \quad [\Delta i_{f}] = C_{EM3} \Delta x_{\psi}. \tag{5}$$

The analytical expressions for  $C_{EM1}$ ,  $C_{EM2}$  and  $C_{EM3}$  are shown in the Appendix, in (49)–(51).

2) *Motion Equations:* The shaft is modelled by using the following differential equation [18]:

$$\frac{d\omega_r}{dt} = \frac{t_m - t_e}{2H}$$

This equation is linear and can be rewritten as [18]:

$$\left[\Delta \dot{\omega}_{sg}\right] = B_{S1} \left[\Delta t_m\right] + B_{S2} \left[\Delta t_e\right]. \tag{6}$$

*3) Exciter:* The exciter includes an alternator in series with a diode rectifier [21]. The model AC5A has been used [23]:

$$\frac{dv_f}{dt} = \frac{1}{T_E}v_r - \frac{S_E + K_E}{T_E}v_f.$$
(7)

This model can be linearised, yielding:

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$$\begin{bmatrix} \Delta \dot{v}_f \end{bmatrix} = A_E \begin{bmatrix} \Delta v_f \end{bmatrix} + B_{E1} \begin{bmatrix} \Delta v_r \end{bmatrix}, \tag{8}$$
$$\begin{bmatrix} \Delta v_f \end{bmatrix} = C_E \begin{bmatrix} \Delta v_f \end{bmatrix}. \tag{9}$$

4) Engine: The dynamics of the diesel engine used as prime mover is modelled as a first-order non-linear system, as it is commonly done in the literature [20]:

$$\frac{dt_m}{dt} = \frac{1}{\tau_m(\sigma + t_m)}(f - t_m).$$
(10)

The linearised model becomes (details in the Appendix):

$$\begin{bmatrix} \Delta \dot{t}_m \end{bmatrix} = A_D \begin{bmatrix} \Delta t_m \end{bmatrix} + B_D \begin{bmatrix} \Delta f \end{bmatrix}, \tag{11}$$

$$\left[\Delta t_{m}\right] = C_{D} \left[\Delta t_{m}\right]. \tag{12}$$

5) Automatic Voltage Regulator (AVR): For simplicity, a proportional-integral (PI) controller has been adopted for the AVR [20]. This regulator controls the amplitude of the PCC voltage  $(v_{sg})$ ,  $(v_{sg} = v_{pcc})$  by generating the set point for the regulated voltage  $(v_r)$ . The module of the PCC voltage is:

$$v_{sg} = \sqrt{v_{sg}^{D^2} + v_{sg}^{Q^2}}.$$
 (13)

The linearised model of the AVR is (details in the Appendix):

$$\begin{bmatrix} \Delta \dot{\alpha} \end{bmatrix} = B_{AVR1} \begin{bmatrix} \Delta v_{sg}^* \end{bmatrix} + B_{AVR2} \begin{bmatrix} \Delta v_{sg}^{DQ} \end{bmatrix},$$
(14)  
$$\begin{bmatrix} \Delta v_r \end{bmatrix} = C_{AVR} \begin{bmatrix} \Delta \alpha \end{bmatrix} + D_{AVR1} \begin{bmatrix} \Delta v_{sg}^* \end{bmatrix} + D_{AVR2} \begin{bmatrix} \Delta v_{sg}^{DQ} \end{bmatrix}.$$
(15)

6) Speed Governor: The speed governor controls the SG frequency by modifying the fuel injection set-point. For simplicity, a PI controller has been used [20]. This controller is linear, so state-space matrices can be defined as:

$$\left[\Delta\dot{\beta}\right] = B_{Gov} \left[\Delta\omega_r^*\right] + B_{Gov2} \left[\Delta\omega_r\right],\tag{16}$$

$$\left[\Delta v_r\right] = C_{Gov} \left[\Delta\beta\right] + D_{Gov} \left[\Delta\omega_r^*\right] + D_{Gov} \left[\Delta\omega_r\right].$$
(17)

7) Diesel Generator Aggregated Small-Signal Model: The detailed models defined in the previous subsections can be merged together to form the model of the diesel generator.



Fig. 3. SRF-PLL diagram of control in (dq).

Input variables are transformed from real to pu values, while the output variables are transformed from pu to real values. The resulting model is:

$$\begin{bmatrix} \Delta \dot{x}_{SG} \end{bmatrix} = A_{SG} \begin{bmatrix} \Delta x_{SG} \end{bmatrix} + B_{SG1} \begin{bmatrix} \Delta v_{sg}^* \end{bmatrix} + B_{SG2} \begin{bmatrix} \Delta \omega_{sg}^* \end{bmatrix} + B_{SG3} \begin{bmatrix} \Delta v_{sg}^{DQ} \end{bmatrix}, \quad (18)$$

$$\left[\Delta i_s^{DQ}\right] = C_{SG1} \left[\Delta x_{SG}\right], \quad \left[\Delta \omega_{sg}\right] = C_{SG2} \left[\Delta x_{SG}\right]. \tag{19}$$

## B. Small-Signal Model of the GFe-VSC

This section presents the small-signal model for the GFe-VSC [24]. It includes an LCL filter, a current controller (in dq), the calculation of the current set-points and a PLL. The detailed state space model can be found in the Appendix and in the literature [24]. Only the PLL is presented here.

1) Phase-Locked Loop (PLL): A PLL is used to synchronise the GFe-VSC (in dq) with the voltage of the PCC (in DQ), as shown in Fig. 3. A typical PLL implementation has been considered [7, 24]:

$$\dot{\epsilon} = k_{ipll} (v_{pcc}^D \cos \delta_{PLL} + v_{pcc}^Q \sin \delta_{PLL}), \qquad (20)$$
$$\dot{\delta}_{PLL} = \epsilon + k_{ppll} (v_{pcc}^D \cos \delta_{PLL} + v_{pcc}^Q \sin \delta_{PLL}) - \omega_{sg}. \qquad (21)$$

Clearly, the PLL is a non-linear system as it has trigonometric functions involved. The linearised model is:

$$\begin{bmatrix} \Delta \dot{\epsilon} \\ \Delta \dot{\delta}_{PLL} \end{bmatrix}_{i} = A^{i}_{PLL} \begin{bmatrix} \Delta \epsilon \\ \Delta \delta_{PLL} \end{bmatrix}_{i} + B^{i}_{PLL1} \left[ \Delta v^{DQ}_{pcc} \right]_{i} + B^{i}_{PLL2} \left[ \Delta \omega_{com} \right]_{i}, \quad (22)$$

The detailed matrices can be found in the Appendix.

2) Aggregated Small-Signal Model of a GFe-VSC: The small-signal models of the current set-points calculation, the current controller, the LCL filter and the PLL are then combined to obtain the GFe-VSC model:

$$\begin{aligned} \left[\Delta \dot{x}_{inv}\right]_{GFe} &= A^{GFe} \left[\Delta x_{inv}\right]_{GFe} + B^{GFe}_{PQ} \left[\Delta p^*\right]_{GFe} + \\ &+ B^{GFe}_{V} \left[\Delta v^{DQ}_{pcc}\right]_{GFe} + B^{GFe}_{\omega} \left[\Delta \omega_{com}\right]_{GFe}, \end{aligned}$$
(23)

where

$$\begin{split} [\Delta x_{inv}]_{GFe} &= \left[\Delta \epsilon, \Delta \delta_{PLL}, \Delta \gamma^{dq}, \Delta i_i^{dq}, \Delta v_c^{dq}, \Delta i_o^{dq}\right]_{GFe}^T, \\ [\Delta u_{inv}]_{GFe} &= \left[\Delta p^*, \Delta q^*, \Delta v_{pcc}^{DQ}, \Delta \omega_{com}\right]_{GFe}^T, \\ [\Delta i_{DQ}]_{GFe} &= C^{GFe} \left[\Delta x_{inv}\right]_{GFe}. \end{split}$$

One should notice  $v_{pcc}^{DQ}$  represents an input variable of the state-space model. However, in the GFe-VSC model, variables are referenced to the local dq reference frame. Therefore, they should be transformed by taking into account (105), (107)

$$A^{MG1} = \begin{bmatrix} A_{SG} + B_{SG3}D_{R1}C_{SG1} & B_{SG3}D_{R1}C^{GFe} & B_{SG3}D_{R2} \\ B_{\omega}^{GFe}C_{SG2} + B_{V}^{GFe}D_{R1}C_{SG1} & A^{GFe} + B_{V}^{GFe}D_{R1}C^{GFe} & B_{V}D_{R2} \\ B_{L2}C_{SG2} + B_{L1}D_{R1}C_{SG1} & B_{L1}D_{R1}C^{GFe} & A_{L} + B_{L1}D_{R2} \end{bmatrix}$$
$$A^{MG2} = \begin{bmatrix} A^{GFe} + B_{\omega}^{GFo}C_{\omega}^{GFo} + B_{V}^{GFo}D_{R1}C_{I}^{GFe} & B_{V}^{GFo}D_{R1}C_{GFe} \\ B_{\omega}^{GFe}C_{\omega}^{GFo} + B_{V}^{GFe}D_{R1}C_{GFe}^{GFe} & A^{GFe} + B_{V}^{GFe}D_{R1}C_{GFe} \\ B_{L2}C_{\omega}^{GFo} + B_{L1}D_{R1}C_{I}^{GFo} & B_{L1}D_{R1}C^{GFe} & A_{L} + B_{L1}D_{R2}C_{L} \end{bmatrix}$$

and (108), derived from (1). This transformation is directly included in the state-space matrices  $A^{GFe}$ ,  $B_{PQ}^{GFe}$ ,  $B_{V}^{GFe}$ ,  $B_{\omega}^{GFe}$ , and  $C^{GFe}$ , in (109)–(113).

# C. Small-Signal Model of the GFo-VSC

The model of the GFo-VSC consist of the models of the LCL filter, the current controller, the voltage controller and the droop controller (details in the Appendix) [19].

1) Aggregated Small-Signal Model of the GFo-VSC: All the models mentioned above are combined to obtain the small-signal model of the GFo-VSC:

$$\begin{aligned} \left[\Delta \dot{x}_{inv}\right]_{GFo} &= A^{GFo} \left[\Delta x_{inv}\right]_{GFo} + B^{GFo}_{In} \left[\frac{\Delta \omega_{nd}}{\Delta v_{nd}^{dq}}\right]_{GFo} + \\ &+ B^{GFo}_{V} \left[\Delta v^{DQ}_{pcc}\right]_{GFo} + B^{GFo}_{\omega com} \left[\Delta \omega_{com}\right]_{GFo}, \end{aligned}$$
(24)

$$\left[\Delta i_o^{DQ}\right]_{GFo} = C_I^{GFo} \left[\Delta x_{inv}\right]_{GFo}, \qquad (25)$$

$$[\Delta\omega_{com}]_{GFo} = C_{\omega}^{GFo} [\Delta x_{inv}]_{GFo} + D_{\omega}^{GFo} \begin{bmatrix} \Delta\omega_{nd} \\ \Delta v_{nd}^{dq} \end{bmatrix}_{GFo},$$
(26)

where

$$\begin{split} [\Delta x_{inv}]_{GFo} &= \\ &= \left[\Delta \delta, \Delta \tilde{P}, \Delta \tilde{Q}, \Delta \phi^{dq}, \Delta \gamma^{dq}, \Delta i_i^{dq}, \Delta v_c^{dq}, \Delta i_o^{dq}\right]_{GFo}^T. \end{split}$$

As in (23), the state-space matrices include the transformations between the reference frames (details in the Appendix).

#### D. Load Modelling

A resistive-inductive linear load has been considered [18]:

$$\left[\Delta i_l^{DQ}\right] = A_L \left[\Delta i_l^{DQ}\right] + B_{L1} \left[\Delta v_{pcc}^{DQ}\right] + B_{L2} \left[\Delta \omega_{com}\right].$$
(27)

The state matrices can be found in the Appendix.

# E. Coupling of Devices

The linearised mathematical models of the diesel gen-set, the converters and the load include the variable  $\Delta v_{pcc}^{DQ}$  as an input and the current as an output. Therefore, an auxiliary resistor has been used to define this voltage adequately ( $R_x$ in Fig. 1) [19]:

$$\begin{bmatrix} \Delta v_{pcc}^{DQ} \end{bmatrix} = D_{R1} \begin{bmatrix} \Delta i_o^{DQ} \end{bmatrix}_{GFo} + D_{R1} \begin{bmatrix} \Delta i_o^{DQ} \end{bmatrix}_{GFe} + D_{R2} \begin{bmatrix} \Delta i_l^{DQ} \end{bmatrix}.$$
(28)

# F. Aggregated Model of MG1: Gen-Set, GFe-VSC and load

Because of the MG topology  $v_{pcc}^{DQ}$  and  $\omega_{com}$  are equivalent to  $v_{sg}^{DQ}$  and  $\omega_{sg}$  (their real values). Then, the MG model is:

$$\begin{bmatrix} \Delta \dot{x}_{SG} \\ \Delta \dot{x}_{inv} \\ \Delta \dot{x}_{L} \end{bmatrix} = A^{MG1} \begin{bmatrix} \Delta x_{SG} \\ \Delta x_{inv} \\ \Delta x_{L} \end{bmatrix} + B_{v}^{MG1} [\Delta v_{sg}^{*}] + B_{PQ}^{MG1} \begin{bmatrix} \Delta p^{*} \\ \Delta q^{*} \end{bmatrix}, \quad (29)$$

where

$$B_v^{MG1} = \begin{bmatrix} B_{SG1} \\ 0 \\ 0 \end{bmatrix}, B_\omega^{MG1} = \begin{bmatrix} B_{SG2} \\ 0 \\ 0 \end{bmatrix}, B_{PQ}^{MG1} = \begin{bmatrix} 0 \\ B_{PQ}^{GFe} \\ 0 \end{bmatrix}$$

G. Aggregated Model of MG2: GFo-VSC, GFe-VSC and Load In MG2,  $v_o^{DQ} = v_{pcc}^{DQ}$ . By using a virtual resistor to combine all the elements, the following model is obtained:

$$\begin{bmatrix} \Delta \dot{x}_{invGFo} \\ \Delta \dot{x}_{invGFe} \\ \Delta \dot{x}_L \end{bmatrix} = A^{MG2} \begin{bmatrix} \Delta x_{invGFo} \\ \Delta x_{invGFe} \\ \Delta x_L \end{bmatrix} + B^{MG2}_{\omega v} \begin{bmatrix} \Delta \omega_{nd} \\ \Delta v_{nd}^{dq} \end{bmatrix} + B^{MG2}_{PQ} \begin{bmatrix} \Delta p^* \\ \Delta q^* \end{bmatrix}, \quad (30)$$

where

$$B_{\omega v}^{MG2} = \begin{bmatrix} B_{In}^{GFo} + B_{\omega com}^{GFo} \begin{bmatrix} D_{P\omega}^{GFo} & 0 \end{bmatrix} \\ B_{\omega}^{GFe} \begin{bmatrix} D_{P\omega}^{GFo} & 0 \end{bmatrix} \\ B_{L2} \begin{bmatrix} D_{P\omega}^{GFo} & 0 \end{bmatrix} \end{bmatrix}, B_{PQ}^{MG2} = \begin{bmatrix} 0 \\ B_{PQ}^{GFe} \\ 0 \end{bmatrix}.$$

# IV. ANALYTIC STUDY

In this section, MG1 and MG2 are analysed. System stability is analysed based on the information obtained from the system eigenvalues and the participation factors (Fig. 4 and Fig. 5 for MG1, and Fig. 6 and Fig. 7 for MG2, respectively) [18]. The small signal model of each scenario presented in the paper was developed considering their individual operating points. The operating point of each scenario was calculated by means of a non-linear model developed in Simulink. Then, the small signal model was used to calculate eigenvalues and participation factors. Participation factors have been calculated as [18]:

$$p_{ki} = \frac{\delta \lambda_i}{\delta a_{kk}} = \psi_{ik} \phi_{ki}, \qquad (31)$$

where  $\psi_{ik}$  and  $\phi_{ki}$  represent the k element of the left and right i eigenvectors, respectively. Eigenvectors were normalized so that  $\psi_i \phi_i = 1$ . This implies that the sum of the participation factors along a row or a column is equal to one:

$$\sum_{i=1}^{n} p_{ki} = 1, \quad \sum_{k=1}^{n} p_{ki} = 1.$$
(32)

#### A. Test System Description

The nominal power of the SG is 60 kVA. The electromechanical parameters of the SG and the diesel engine are taken

Par.	Val.	Par.	Val.	Par.	Val.
$r_a$	3.54E - 2	$l_{kkq}$	1.4967	σ	1
$r_{f}$	6E-4	$l_{ad}$	2.38	$k_{pAVR}$	2.47
$r_{kd}$	3.54E - 2	$l_{aq}$	1.24	$k_{iAVR}$	0.42
$r_{kQ}$	4.28E - 2	H	3.704E - 1	$k_{pGov}$	5.57
$l_d$	2.83	$T_E$	0.8	$k_{iGov}$	6.39
$l_q$	1.69	$S_E$	1	Poles	4
$l_{ff}$	2.6371	$K_E$	0.68	$U_B$	$400\sqrt{2}$
$l_{kkd}$	2.58	$ au_m$	0.1	$f_B$	50Hz

TABLE I Diesel Genset Parameters (SC1).

TABLE II GF0 and GFe-VSC parameters (SC1).

Param	Val	Param	Val
$L_i$	2.3 mH	$k_{iv}(GFo)$	6.92
$R_i$	72.2 mΩ	$m_p$	6.67E-6
Lo	0.93 mH	$n_q$	3.33E-4
$R_o$	29.2 mΩ	$\omega_c$	31.41
$C_d$	8.83 μF	$k_{pc}(GFe)$	4.43
$k_{pc}(GFo)$	1.81	$k_{ic}(GFe)$	3.05E3
$k_{ic}(GFo)$	2.17E3	$k_{ppll}$	0.0136
$k_{pv}(GFo)$	0.0553	$k_{ipll}$	0.0492

from reference [20]. For the excitation system, the reference values are taken from the AC5A model in [23]. GFo- and GFe-VSC parameters are defined in Table II. Two VSCs of the same power rating (15kVA) (and, therefore, the same *LCL* parameters) have been considered, because the converter power ratio introduces a key restriction during the PLL design.

A base scenario (SC1) has been defined for both MGs. In SC1, the load is consuming 18 kW. The active power setpoint for the GFe-VSC is 4 kW and the reactive power setpoint is zero. The voltage and frequency set points of the SG are 400 V and 50 Hz, respectively. The AVR and the speed governor have been designed to be as fast as possible, resulting in bandwidths of 0.17 Hz and 1.7 Hz, respectively (see Table I). Controllers have been designed by using openloop techniques, with phase margins of 45 deg (speed governor and current controllers), 50 deg (AVR) and 60 deg (PLL and voltage controller). The bandwidth of the current controller and the PLL of the GFe-VSC are set to 490 Hz and 1.4 Hz, respectively, while the bandwidth of the current and voltage controllers of the GFo-VSC are set to 490 Hz and 30 Hz, respectively. The rest of parameters are defined according to Table II. The additional scenarios considered in the paper are defined in Table III. Scenarios SC2 to SC23 are derived by modifying some parameters of a base scenario.

#### B. Participation Factors for MG1 (Diesel Gen-Set)

Fig. 5 shows the modulus of the participation factors for MG1, SC1. For instance, the modulus of the participation factors of state  $\delta_{PLL}$  in the modes 7-13 are 0.03, 0.03, 0.7, 0.7 0.08, 0.08 and 0.03, respectively. Modes are ordered by numbers in the *x*-axis according to their proximity to the imaginary axis (left meaning closer to it). The states related to

TABLE III DEFINITION OF SCENARIOS SC2-SC23 SHOWING THE INITIAL (BASE) SCENARIO, THE PARAMETER CHANGED AND ITS NEW VALUE.

SC	Base SC	Changed Param	New Value
SC2	SC1	$P_{GFe}$	6 kW
SC3	SC1	$P_{GFe}$	9 kW
SC4	SC1	$P_{GFe}; P_L$	2,6 kW; 12 kW
SC5	SC1	$P_{GFe}; P_L$	4 kW; 12 kW
SC6	SC1	$P_{GFe}; P_L$	6 kW; 12 kW
SC7	SC6	$BWGFe_{cc}$	380 Hz
SC8	SC6	$BWGFe_{cc}$	260 Hz
SC9	SC7	J	3,6 kg $m^2$
SC10	SC7	J	$0,9 \mathrm{~kg}~m^2$
SC11	SC7	BWf	2,1 Hz
SC12	SC7	BWf	0,5 Hz
SC13	SC1	$P_{GFe}; P_L$	12 kW; 18 kW
SC14	SC1	$P_{GFe}; P_L$	8 kW; 12 kW
SC15	SC14	$BWGFe_{cc}$	380 Hz
SC16	SC14	$BWGFe_{cc}$	260 Hz
SC17	SC16	$GFo_{vc}; GFo_{cc}$	31 Hz; 260 Hz
SC18	SC16	$GFo_{vc}; GFo_{cc}$	20 Hz; 490 Hz
SC19	SC16	$GFo_{vc}; GFo_{cc}$	20 Hz; 260 Hz
5020	SC1	$P_{GFe}; P_L;$	9 kW; 23 kW;
SC20	301	$GFe_{cc}; GFe_{PLL}$	120 Hz; 17 Hz
		$P_{GFe}; P_L;$	11 kW; 13 kW;
SC21	SC1	$GFo_{cc}; GFo_{vc};$	520 Hz; 34 Hz;
		$GFe_{cc}; GFe_{PLL}$	250 Hz; 19 Hz
SC22	SC20	$GFe_{cc}; GFe_{PLL}$	380 Hz; 7 Hz
		$GFo_{cc}; GFo_{vc};$	380 Hz; 40 Hz;
SC23	SC21	$GFe_{cc}; GFe_{PLL}$	380 Hz; 7 Hz

the SG are mainly affected by Modes 1–5. These modes have limited influence on the the PLL states, while their influence on the rest of the GFe states is negligible. Modes 6–12 participate in the engine and the rotor states ( $\omega_r$  and  $t_m$ ), but also in the PLL states ( $\epsilon$  and  $\delta_{PLL}$ ), thus creating a dynamic interaction between the two devices. Modes 12–13 are related to the PLL and the current controller and, therefore, represent their interaction. The rest of the modes are mainly related to electromagnetic phenomena and represent the link between the current controller, the *LCL* filter variables and the load.

The interaction between  $\delta_{PLL}$  and the mechanical states is stronger especially for the reduced bandwidth of the speed governor loop. Additionally, the states  $\omega_r$ ,  $t_m$  and  $\gamma_d$  show a stronger coupling with  $\delta_{PLL}$  when the bandwidths of the PLL and the current controller decrease, and also when the inertia of the generator is reduced. The states  $\delta_{PLL}$  and  $\gamma_d$ also show a stronger interaction when the power injected by the GFe-VSC increases. No relevant interactions were observed between the PLL and the voltage controller of the diesel gen-set, as it was previously reported in the literature when standard excitation AC4A was used [14, 23]. This could be motivated by the use of different AVR and excitation system [25].

From these results, it is clear that the PLL participates in the mechanical states of the MG. This will be studied in the next subsection where the system eigenvalues will be analysed.



Fig. 4. Trajectory of MG1 eigenvalues (low frequency) for different scenarios when the bandwidth of PLL is increased. (a) GFe-VSC injecting 22%, 33% and 50% of the load, which takes two values (12 kW and 18 kW), (b) current controller bandwidths of GFe-VSC: 490 Hz, 380 Hz and 260 Hz and (c) different designs of speed governor and SG inertia. See Table III for detailed definition of scenarios.



Fig. 5. Participation matrix for MG1, SC1. Eigenvalues are ordered left to right according to their distance from the imaginary axis.

#### C. Stability Limits of MG1 (Diesel Gen-Set)

Fig. 4 shows how the eigenvalues of MG1 are affected by the PLL bandwidth in various scenarios. Only low frequency eigenvalues are shown because PLL mainly affects low frequency dynamics. Fig. 4 (a) considers six different loading scenarios (SC1-SC6), with three different levels of power injected by GFe-VSC (22 %, 33 % and 50 % of load). The eigenvalues that are more affected by the PLL move towards the imaginary axis when the relative power share of the GFe-VSC increases. This effect is more pronounced and evident at light load conditions. For instance, in both SC1 and SC4 Gfe-VSC has a 22 % power share but the eigenvalues in SC4 are located closer to the imaginary axis. This will result in a reduction of the stability limits linked with the parameters of the PLL.

In Fig. 4 (b) three scenarios with different bandwidths of the current controller are studied (SC6, SC7 and SC8). The current controller bandwidth has a strong influence on the maximum bandwidth achievable by the PLL. For instance, for a current controller bandwidth of 260 Hz (SC8), the MG becomes unstable when the PLL bandwidth is higher than

# 27 Hz.

Different scenarios for the inertia and the design of the speed governor have been examined and the results shown in Fig. 4 (c) (SC7, SC9, SC10, SC11 and SC12). It shows that the SG inertia and the bandwidth of the speed governor have little influence on the eigenvalues linked with PLL. This interaction is even weaker when the PLL bandwidth increases.

Summarising, for the type of generator, prime movers and controllers considered, the mechanical parameters have only a small contribution to the eigenvalues related to the PLL. In contrast, the loading conditions and the design of the current controller greatly affect the aforementioned eigenvalues. With this respect, high levels of load, low relative power contribution from the GFe-VSC and fast current controllers are preferable. A summary is presented in Table IV.

#### D. Participation Factors for MG2 (GFo-VSC)

Fig. 7 shows the participation matrix for MG2 for SC1. All the modes of MG2 mainly participate in one state (or in a couple of related states, like  $\epsilon$  and  $\delta_{PLL}$ ). Clearly, the states and modes are more decoupled than in MG1. It can be seen that Modes 2–3 mainly participate in the states related to the PLL, and to a lesser extent, to the voltage controller of the GFo-VSC. The interaction between these parameters is studied in the following section.

# E. Stability Limits of MG2 (GFo-VSC)

Fig. 6 shows the trajectory of the low frequency eigenvalues of MG2 for the increase of the PLL bandwidth, for different scenarios. Fig. 6 (a) shows that in MG2, as it happened in MG1, the eigenvalues move to the right when the power injected by the GFe-VSC increases. Scenarios with light load conditions (SC5, SC6 and SC14) also tend to destabilise the system compared to those with more load (SC2, SC63 and SC13 respectively).

Fig. 6 (b) shows that eigenvalues move towards the imaginary axis if the bandwidth of the current controller decreases, as it happened in MG1. This may result on limited PLL bandwidth for some loading conditions.

Fig. 6 (c) shows MG2 eigenvalues when the bandwidth of the PLL increases for different designs of the GFo-VSC voltage and current controllers. As shown in Fig. 6 (c), the



Fig. 6. Trajectory of low frequency eigenvalues of MG2 for different scenarios when the bandwidth of PLL is increased. (a) GFe-VSC injecting 33%, 50% and 67% of power load that takes 2 values (12kW and 18kW), (b) GFe-VSC current controller bandwidths: 490Hz, 380Hz and 260Hz and (c) different designs of the GFo-VSC voltage and current controllers. See Table III for detailed definition of scenarios.



Fig. 7. Participation matrix of MG2, SC1. Eigenvalues are ordered left to right according to their distance from the imaginary axis.

eigenvalues are closer to the imaginary axis when the voltage controller is slowed down (SC18 and SC19). In general, interactions between the voltage and current controllers are avoided by ensuring a sufficient time-constant separation between them. More precisely, current controllers are typically designed to be faster than voltage controllers. However, as shown in Fig. 6 (c), increasing the current controller bandwidth has a negative effect on the stability (SC18), unless the voltage controller bandwidth is increased at the same time (SC16).

Summarising, loading conditions and the design of the GFe-VSC current controller have impact on the eigenvalues related to the PLL, just as it happened in MG1. High load, low relative power injection by the GFe-VSC and fast current controller for the GFe-VSC allow higher PLL bandwidths. The GFo-VSC also affects the PLL design. The PLL bandwidth can be increased with fast voltage controllers and a slow current controllers. In this case the loading conditions do not limit the PLL design as much as in the case of MG1. These aspects are summarised in Table IV.

TABLE IV CHANGES THAT REDUCE PLL BANDWIDTH STABILITY LIMITS.

Parameter	Type of change in parameter	
$P_L$	Decrease ↓	
$P_{GFe}/P_{GFo}$	Increase $\uparrow$ (specially in MGs with SG)	
$BWGFe_{cc}$	Decrease ↓	
$BWGFo_{vc}$	Decrease ↓	
$BWGFo_{cc}$	Increase ↑	

# V. GUIDELINES FOR THE DESIGN OF CONTROL PARAMETERS IN A MG

The conclusions drawn from the analysis have been used to develop and propose design guidelines for the selection of control parameters in a MG. The guidelines assume the use of the small-signal model, however, some of the recommendations are generally applicable and are helpful even without the model. The flowchart with the design procedure is shown in Fig. 8. The boxes of the flowchart also indicate the section in which the task is addressed. The design of the parameters is based on the stability limits calculated using the smallsignal model. Initially, a stable scenario is used by applying typical values for the controllers. Preliminary stability limits are calculated by assuming this scenario, that is also used to compute a stable operating point. Then, the preliminary stability limits are used to define the controllers of the MG under most unfavourable conditions for the stability (low levels of power injected by the GFo device and high levels of power injected by the GFe-VSC). Both non-linear and linear models are developed for the unfavourable condition scenario. MG controllers are redesigned to meet the specifications taking into account the stability limits under the unfavourable conditions.

#### A. Preliminary Controller Settings

1) Diesel-Based MGs: To start with, the controllers of diesel gen-sets can be configured to be as fast as it is required while the bandwidth of the PLL should be similar to that of the frequency controller. The current controller of the GFe-VSC should be designed in a typical way by using the desired settling time and the adequate damping of the LCL filter resonance [6,7]. The bandwidth of the dc-voltage controller should not be close to that one used for the PLL to avoid



Fig. 8. Flow chart of the proposed method for the design of MG controllers.

undesired oscillations. Also, it should not be unnecessarily wide [16, 26].

2) Converter-Based MGs: The current controllers of the GFe-VSC and the GFo-VSC should be designed by using standard design rules [6,7]. Then, the voltage controller of the GFo-VSC should be configured to be as fast a possible, yet it needs to guarantee a sufficient time-constant separation between the current and the voltage loops. The PLL can be designed to be fast enough to track standard frequency variations in power grids [27] (a bandwidth between 1 and 10 Hz would work in most cases). The dc-voltage controller, if required, should be designed in a similar way to the case of diesel-based MGs.

# B. Model Development, Operating Points and Stability

1) Non-Linear Model and Operating Point: A non-linear model of the microgrid is used to perform the time-domain

simulations and to calculate the operating point. The model should include all the non-linear equations of the desired GFo device (SG or VSC), the GFe and the load. In order to find a suitable operating condition to start the analysis and the control design, it is recommended to set the GFo to feed a moderate load (50-80 % of its rated power) and the GFe not to inject any power.

2) Small-Signal Model of the MG: A small-signal model of the MG should be developed. It can be obtained by using the analytical expressions included in this paper (in Section III), or by using automatic tools (e.g. Simulink Control Design toolbox [28]).

3) Initial Conditions for the Stability Test: Stability limits of the bandwidth of MG controllers can be found by using the small-signal analysis techniques presented in the article. These limits will be used for the design of MG parameters under most unfavourable conditions.

# C. Fine Tuning of Control Parameters under Unfavourable Conditions

An unfavourable operating condition can be found when the GFo injects its minimum power and when the GFe relative power injection is high. By following the steps mentioned before, the designer may have already found control parameter values that guarantee the stability. However, a more elaborated selection can lead to improved stability margins and an optimisation procedure can be used to find the optimal selection.

1) MGs Formed by Diesel Gen-Sets: The PLL can be redesigned by taking into account the upper limit of its bandwidth and the expected frequency variations. If the design specifications cannot be met, the GFe-VSC current controller should be made faster to increase the stability limit of the PLL bandwidth.

2) MGs Formed by GFo-VSCs: The GFe-VSC current controller can be redesigned by taking into account the lower stability limit imposed by the MG and other restrictions imposed by the *LCL* resonance and the switching frequency. Finally, the PLL can be redesigned by taking into account its upper stability limit and the expected variations of the MG frequency. If the design specifications cannot be fulfilled, the voltage and current controllers should be redesigned by reducing the bandwidth of the GFo-VSC current controller and increasing both - the bandwidth of the GFe-VSC current controller of the GFo-VSC to allow for wider PLL bandwidths.

#### D. Stability Analysis under Unfavourable Conditions

It is recommended to calculate the small-signal model for the operation under these conditions. Also, it is recommended to carry out transient simulations in order to check if the system operates within its limits even during large transients. If the system is not well damped (e.g. damping factor smaller than 0.3), the controllers should be redesigned. This can be checked by analysing the system eigenvalues. For example, the PLL and the current controller of the GFo-VSC can be slowed down, while the speed of the current controller of the GFe-VSC and the voltage controller of the GFo-VSC can be increased.



Fig. 9. Lab photo. From left to right: (green) 75 kVA converter used as emulator for the SG and part of the load in MG1, (cyan) real time targets (converter controllers), (blue) 15 kVA converter used as GFo-VSC, (red) 15 kVA converter used as GFe-VSC, (yellow) ac bus bars and (magenta) loads.



Fig. 10. Electrical diagram of the laboratory. The common elements of MG1 and MG2 are shown in red (GFe-VSC and load). Grid emulator is shown in green, and GFo-VSC is shown in blue. MG1: red and green. MG2: red and blue.

#### VI. EXPERIMENTAL VALIDATION

#### A. Prototype Description

MG1 and MG2 were implemented in the Smart Energy Integration Lab (SEIL) [29, 30] (Fig. 9 and Fig. 10). A 75 kVA VSC was used as the grid emulator. It used model-based approach to emulate the operation of the diesel gen-set and part of the load (12 kW) in MG1. Two 15 kVA converters were used as GFo-VSC and GFe-VSC in the experimental validation. Different loading conditions were implemented by using a variable, progammable resistive load bank. The singleline electrical diagrams of the two MG implementations are shown in Fig. 10 (MG1 in red and green and MG2 in red and blue).

#### B. Validation of Small-Signal Models

Fig. 11 and Fig. 12 show a comparison between the transient responses obtained from the experimental setup and the linearised models when applying SC1. Since MG dynamics is slow, the models are validated by using dq signals, as it is common in the literature [19, 24]. Experimental dq signals were obtained from the control boards of the converters, without filters. The experimental results were also obtained by using an oscilloscope, as shown in Fig. 13. However, in this case, it was difficult to compare the theoretical results with the experimental waveforms. A 10 V and a 0.2 Hz step were applied to the set points of the SG voltage and frequency



Fig. 11. Response of MG1 when there is step change of (a) 10 V and (b) 0.2 Hz in the SG voltage and frequency set-points. (red) Experimental and (blue) linearised model results.



Fig. 12. Response of MG2 for (a) a step of 10 V applied to the nominal voltage of the droop controller and (b) a step of 0.2 Hz applied to the nominal frequency of the controller. (red) Experimental results and (blue) linearised model results.

controllers (Fig. 11). The same steps were used for the GFo-VSC (Fig. 12). Clearly, the small-signal models accurately reproduce the low-frequency dynamics of the experimental platform. Since high frequency phenomena (e.g. pulse-width modulation) was not considered in the modelling, high frequency dynamics cannot be predicted by using the developed linear models. In  $v_{dq}$ ,  $\omega_{GFe}$  and  $\omega_{GFo}$  some harmonics can be observed due to the lack of filters at the measuring stage. This oscillations are not present in  $\omega_{SG}$  because this variable is taken from the emulated model.

#### C. Validation of Stability Limits

The theoretical stability limits of the PLL were validated experimentally. MG1 was validated for SC20, where the load was 23 kW (12 kW were emulated). The GFe-VSC injected 9 kW (39 %). The SG parameters are defined in Table I. The current controller bandwidth was 120 Hz. In this scenario, the linear model analyses suggested a maximum PLL bandwidth



Fig. 13. Response of modulus of  $v_{abc}$ , when there is a 10 V step change of the SG set-point. Horizontal scale, 1 s/div, vertical scale, 20 V/div.



Fig. 14. Response of the MGs in SC20 and SC21, respectively, when the load decreases 1kW (and hence the ratio of power injected by the GFe-VSC increases) making both MGs unstable.



Fig. 15. Response of MG1 in SC22 when the load decreased 1kW with a recommended controller design. Comparison between the time responses obtained from the corresponding eigenvalues and the experimental setup.

of 18 Hz. However, during the validation, the system remained stable up to a bandwidth of 17 Hz. Even though the dynamics was accurately described by the linear model, the range of frequency for the system to be marginally stable differs slightly. This is common in experimental validations as linear models show different sensitivity to parameter variation at the edge of the stability region [31]. However, small-signal tools are still useful in grid applications since the main objective here is to ensure that the system operates away from the unstable region [18]. Fig 14 (a) shows the response of MG1 when the PLL bandwidth was 17 Hz and a variation of the load from 23 kW (SC20) to 22 kW was applied. This means that the GFe-VSC power varies from 39 % to 40 % of the total demand. The MG becomes unstable due to the higher ratio of power injected by the GFe-VSC. At t = 10 s and t = 12 s the load is increased again and the MG slowly recovers its stable operation. The stability limits of MG2 where validated in SC21:  $P_{load} = 13$  kW,  $P_{GFe-VSC} = 11$  kW,  $BW_{ccGFo} =$ 520 Hz,  $BW_{vcGFo} = 34$  Hz,  $BW_{ccGFe} = 250$  Hz. The linear model analysis suggests the MG becomes unstable for PLL bandwidths higher than 21 Hz. In the validation, the maximum



Fig. 16. Response of MG2 in SC23 when the load decreased 1 kW, with the fine tuning. Comparison between the time responses obtained from the corresponding eigenvalues and the experimental setup.

bandwidth was 19 Hz. Fig 14 (b) shows the response of MG2 in SC21 when there was a load variation from 13 kW to 12 kW (85 % and 92 % of the load power injected by the GFe-VSC). MG2 became unstable with the load step at t = 2 s. The load returned to 13 kW at t = 14 s, and then the system regained its stability.

#### D. Validation of Design Procedure for Control Parameters

Controllers of MG1 and MG2 for SC20 and SC21 were redesigned following the guidelines introduced in Section V-C yieldind SC22 and SC23 respectively. In MG1, the current controller and the PLL of the GFe-VSC were designed to have bandwidths 380 Hz and 7 Hz, respectively. In MG2, the current and voltage controller bandwidths of the GFo-VSC were readjusted to 380 Hz and 40 Hz, respectively. Meanwhile, the bandwidths of PLL and the current controller in GFe-VSC were increased to 380 Hz and 7 Hz, respectively.

Fig. 15 (c) and (d) and Fig. 16 (c) and (d) show the dynamic response of the MGs for SC22 and SC23, respectively, when the load changes were applied as in Fig. 14. In this case, the MGs did not become unstable due to the improved controller design. Fig. 15 (a) and Fig. 16 (a) show the low-frequency eigenvalues for the two MGs at their operating point. The transient responses associated with the pairs of eigenvalues in Fig. 15 (a) and Fig. 16 (a) are depicted in Fig. 15 (b) and Fig. 16 (b). These responses are calculated as:

$$y_i(t) = e^{\lambda_i t}.$$

In MG1, the dynamic response of  $\omega_r$  can be represented as a linear combination of the time characteristics of the dominant eigenvalues 5 - 8, as the participation matrix suggests in Fig. 5. Also the time response of  $v_d$  can be represented as a linear combination of the time characteristics of eigenvalues 9 - 10. Similarly, in MG2 the responses of the voltage and frequency can be represented as a linear combination of the time characteristics of the voltage and frequency can be represented as a linear combination of the time characteristics of eigenvalues 6-7 and 4-5, respectively. These eigenvalues are mainly linked with the states of the voltage controller and the droop control, respectively, as shown in Fig. 7.

#### VII. DISCUSSION

#### A. Controller Design and Interactions

The analysis of participation factors confirmed that lowfrequency modes are affected by the PLL states and, simultaneously, by the mechanical states of the diesel gen-set and the states of the current and voltage controllers. The eigenvalue analysis revealed the PLL design is closely linked with the loading conditions and the design of the voltage and current controllers of the VSCs. In general, slow GFe-VSC current controllers, slow GFo-VSC voltage controllers and fast GFo-VSC current controllers limit the range of the PLL bandwidth in which the stability is guaranteed. Previous studies also showed that systems may become unstable when the PLL and the dc-voltage controller have similar bandwidths [16, 17]. Therefore, the PLL design is crucial and critical for the stability of a MG.

#### B. Additional Control Loops

The study did not reveal any relevant interactions between the PLL and the voltage control loop of the SG (excitation and AVR), as it was previously reported in the literature for other types of excitation. The most likely cause is that in these works a static excitation system was used, while in this one it is based on alternating current. Also, as reported in the literature, the dc-voltage controller may interact with the PLL and cause instabilities [10, 16, 17]. In order to study this interaction, a standard dc-voltage controller was implemented in the GFe-VSC [7]. Then, the response of  $v_{dPLL}$  was checked for MG2, for different designs of the PLL and the dc-voltage controller. Fig. 17 shows the response of MG2 at SC21 with PLL bandwidth of (a) 10 Hz and (b) 20 Hz (close to the stability limit of the PLL). Fig. 17 shows that  $v_{dPLL}$  exhibits an oscillation, which appears as a result of the PLL design. When PLL is close to its stability limit, in (b), the oscillation is poorly damped. Also, it can be seen that the design of the dcvoltage controller slightly affects the oscillation. In particular, the amplitude of the oscillation is amplified when the dcvoltage controller and the PLL have the same bandwidths. This result is in agreement with previous studies that show that the interaction between PLL and dc-voltage controller is stronger when they have similar bandwidths [16, 17]. Therefore, it would be of interest to study in detail the cases in which the PLL interacts with other control loops (e.g., the governor of the SG) and also the interaction between dc-voltage controller and the rest of the MG controllers.

# C. Advanced PLL Structures

In order to study the effect of advanced synchronisation techniques, the dual second-order generalised integrator (DSOGI) was implemented to calculate the positive sequence of the PCC voltage in the studied MGs [6]. For MG1 and MG2, the non-linear models were used to obtain the stability limits of the DSOGI-PLL bandwidth under different scenarios. Table V shows a comparison of the obtained stability limits of the PLL bandwidth, with and without the additional DSOGI. Both implementations have similar stability limits, although those obtained with the DSOGI were slightly wider. In some scenarios, like in SC20, the MG is stable for all the values of the PLL bandwidth when the DSOGI is used. Fig. 18 (SC20) shows that MG1 remains stable under load changes if a DSOGI-PLL is used, confirming the results presented in Table V. From these results, it can be seen that advanced



Fig. 17. Response of MG2 ( $v_{dPLL}$ ) when there is a 3 kW change of the load and the dc-voltage controller has different bandwidths. (a) SC21 with PLL bandwidth of 10 Hz and (b) SC21 with PLL bandwidth of 20 Hz (stability limit).

TABLE V STABILITY LIMITS OF PLL BANDWIDTH UNDER DIFFERENT SCENARIOS ("-" MEANS "LIMIT NOT FOUND").



Fig. 18. PCC voltage (in dq) of the non-linear model of MG1 (SC20), when there is a 1kW change of the load, with and without a DSOGI-PLL.

synchronisation techniques may offer extended stability limits, yet they have to be studied in more detail.

#### VIII. CONCLUSION

In this paper the dynamic interactions between a GFe-VSC with PLL and two types of GFo devices (diesel gen-set and droop-controlled VSC) in MGs have been studied. Detailed small-signal models for the two different GFo cases have been derived analytically first and then participation factor and

eigenvalue analyses have been used to study the interactions between PLL and other devices in the MG. All the theoretical developments have been validated experimentally.

The stability studies as well as the proposed design guidelines have been presented in this article to help MG designers understand the interactions between GFe PLL, GFo devices and other elements in MGs and facilitate the correct configuration of the control system parameters. The experimental results and the results obtained from the small-signal models showed a good match. Small deviations were seen only for the parameter values that made the system operate close to the maximum PLL bandwidth allowed. Finally, the experimental results for the control system configured according to the proposed design guidelines confirmed that the dynamic response of MGs can be significantly improved by following the developed design criteria.

In future, it would be of interest to study the effect of different SG voltage control loops (especially of those with static excitation and fast AVR), the impact of more advanced PLL topologies as well as the interactions between the dc-voltage controller and other control loops.

#### REFERENCES

- D. P. Morán-Río, J. Roldán-Pérez, M. Prodanovic, and A. García-Cerrada, "Influence of PLL parameters on small-signal stability of microgrids with synchronous generators," in 2020 IEEE Energy Conversion Congress and Exposition (ECCE), 2020, pp. 2451–2458.
- [2] N. Hatziargyriou, *Microgrids: architectures and control.* John Wiley & Sons, 2014.
- [3] A. Ogunjuyigbe, T. Ayodele, and O. Akinola, "Optimal allocation and sizing of pv/wind/split-diesel/battery hybrid energy system for minimizing life cycle cost, carbon emission and dump energy of remote residential building," *Applied Energy*, vol. 171, pp. 153–171, 2016.
- [4] D. E. Olivares, A. Mehrizi-Sani, A. H. Etemadi, C. A. Canizares, R. Iravani, M. Kazerani, A. H. Hajimiragha, O. Gomis-Bellmunt, M. Saeedifard, R. Palma-Behnke, G. A. Jimenez-Estevez, and N. D. Hatziargyriou, "Trends in microgrid control," *IEEE Trans. on Sm. Gri.*, vol. 5, no. 4, pp. 1905–1919, July 2014.
- [5] M. Farrokhabadi, C. A. Cañizares, J. W. Simpson-Porco, E. Nasr-Azadani, L. Fan, P. A. Mendoza Araya, R. Tonkoski, U. Tamrakar, N. Hatziargyriou, D. Lagos, R. W. Wies, M. Paolone, M. Liserre, L. Meegahapola, M. Kabalan, A. H. Hajimiragha, D. Peralta, M. Elizondo, K. P. Schneider, F. Tuffner, and J. Reilly, "Microgrid stability definitions, analysis, and modeling," IEEE PES Task Force on Microgrid Stability Analysis and Modeling, Technical Report, Apr 2018.
- [6] R. Teodorescu, M. Liserre, and P. Rodriguez, Grid converters for photovoltaic and wind Pow. Sys. Wiley-IEEE Press, 2011.
- [7] A. Yazdani and R. Iravani, Voltage-Sourced Converters in Pow. Sys.: Modeling, Control, and Applications. Wiley, 2010.
- [8] B. Wen, D. Boroyevich, R. Burgos, P. Mattavelli, and Z. Shen, "Analysis of d-q small-signal impedance of grid-tied inverters," *IEEE Trans. on Pow. Elec.*, vol. 31, no. 1, pp. 675–687, 2016.
- [9] J. Z. Zhou, H. Ding, S. Fan, Y. Zhang, and A. M. Gole, "Impact of short-circuit ratio and phase-locked-loop parameters on the small-signal behavior of a VSC-HVDC converter," *IEEE Transactions on Power Delivery*, vol. 29, no. 5, pp. 2287–2296, 2014.
- [10] L. Harnefors, X. Wang, A. G. Yepes, and F. Blaabjerg, "Passivity-Based Stability Assessment of Grid-Connected VSCs—An Overview," *IEEE Journal of Emerging and Selected Topics in Power Electronics*, vol. 4, no. 1, pp. 116–125, 2016.
- [11] F. Katiraei, M. R. Iravani, and P. W. Lehn, "Small-signal dynamic model of a micro-grid including conventional and electronically interfaced distributed resources," *IET Gen, Tran. & Dis.*, vol. 1, no. 3, pp. 369–378, 2007.
- [12] F. Katiraei, "Dynamic analysis and control of distributed energy resources in a micro-grid," Ph.D. dissertation, University of Toronto, 2005.
- [13] X. Tang, W. Deng, and Z. Qi, "Investigation of the dynamic stability of microgrid," *IEEE Trans. on Pow. Sys.*, vol. 29, no. 2, pp. 698–706, 2014.

- [14] Y. Lin, B. Johnson, V. Gevorgian, V. Purba, and S. Dhople, "Stability assessment of a system comprising a single machine and inverter with scalable ratings," in 2017 North American Power Symposium (NAPS), 2017, pp. 1–6.
- [15] J. Guo, T. Chen, B. Chaudhuri, and S. Y. R. Hui, "Stability of isolated microgrids with renewable generation and smart loads," *IEEE Trans. on Sus. En.*, vol. 11, no. 4, pp. 2845–2854, 2020.
- [16] Y. Huang, X. Yuan, J. Hu, and P. Zhou, "Modeling of VSC Connected to Weak Grid for Stability Analysis of DC-Link Voltage Control," *IEEE Journal of Emerging and Selected Topics in Power Electronics*, vol. 3, no. 4, pp. 1193–1204, 2015.
- [17] D. Wang, L. Liang, L. Shi, J. Hu, and Y. Hou, "Analysis of Modal Resonance between PLL and DC-Link Voltage Control in Weak-Grid Tied VSCs," *IEEE Transactions on Power Systems*, vol. 34, no. 2, pp. 1127–1138, 2019.
- [18] P. Kundur, *Power system stability and control.* New York: McGraw-Hill, 1994.
- [19] N. Pogaku, M. Prodanovic, and T. C. Green, "Modeling, analysis and testing of autonomous operation of an inverter-based microgrid," *IEEE Trans. on Pow. Elec.*, vol. 22, no. 2, pp. 613–625, March 2007.
- [20] J. V. Knudsen, "Modeling, Control, and Optimization for Diesel-Driven Generator Sets," Ph.D. dissertation, Aalborg Univ., 2017.
- [21] I. Boldea, *Synchronous Generators*. Taylor & Francis Group, 2006.
   [22] P. C. Krause, O. Wasynczuk, and S. D. Sudhoff, *Analysis of Electric*
- Machinery and Drive Systems, 2nd ed. Wiley-IEEE Press, Feb. 2002. [23] "IEEE 421.5-2016: Recommended Practice for Excitation System Mod-
- els for Power System Stability Studies," IEEE, Standard, Aug 2016.[24] A. Rodríguez-Cabero, M. Prodanovic, and J. Roldán-Perez, "Analysis of dynamic properties of vscs connected to weak grids including the
- effects of dead time and time delays," *IEEE Trans. on Sus. En.*, vol. 10, no. 3, pp. 1066–1075, July 2019.
  [25] A. Monti, F. Milano, E. Bompard, and X. Guillaud, *Converter-Based*
- Dynamics and Control of Modern Power Systems. Academic Press, 2020.
- [26] L. Harnefors, "Implementation of resonant controllers and filters in fixed-point arithmetic," *IEEE Trans. on Ind. Elec.*, vol. 56, no. 4, pp. 1273–1281, 2009.
- [27] "Regulations for grid connection Energinet," https://en.energinet.dk/ Electricity/Rules-and-Regulations/Regulations-for-grid-connection, accessed: 2021-03-25.
- [28] "Simulink control design toolbox," https://es.mathworks.com/products/ simcontrol.html, accessed: 2021-03-31.
- [29] M. Prodanovic, A. Rodríguez-Cabero, M. Jiménez-Carrizosa, and J. Roldán-Pérez, "A rapid prototyping environment for DC and AC microgrids: Smart Energy Integration Lab (SEIL)," in 2017 IEEE Second Int. Conf. on DC Microgrids (ICDCM), June 2017, pp. 421–427.
- [30] F. Huerta, J. K. Gruber, M. Prodanovic, and P. Matatagui, "Powerhardware-in-the-loop test beds: evaluation tools for grid integration of distributed energy resources," *IEEE Industry Applications Magazine*, vol. 22, no. 2, pp. 18–26, March 2016.
- [31] P. Vorobev, P. H. Huang, M. A. Hosani, J. L. Kirtley, and K. Turitsyn, "High-Fidelity Model Order Reduction for Microgrids Stability Assessment," *IEEE Trans. on Pow. Sys.*, vol. 33, no. 1, pp. 874–887, 2018.

#### APPENDIX

# A. State-space matrices of the SG

1) State-space matrices of the electromechanical model: The equations linking DQ stator and rotor flux linkages in a SG with damper windings are [18]:

$$v_{sg}^D = \frac{d\psi^D}{dt} - \omega_r \psi^Q - r_a i^D, \qquad (33)$$

$$v_{sg}^Q = \frac{d\psi^Q}{dt} + \omega_r \psi^D - r_a i^Q, \qquad (34)$$

$$v_f = \frac{d\psi_f}{dt} + r_f i_f,\tag{35}$$

$$0 = \frac{d\psi_k^D}{dt} + r_{kd}i_k^D,\tag{36}$$

$$0 = \frac{d\psi_k^Q}{dt} + r_{kq} i_k^Q, \tag{37}$$

where the stator and rotor flux linkages are defined as [18]:

$$\psi^D = -l_d i^D_{sg} + l_{afd} i_f + l_{akd} i^D_k, \qquad (38)$$

$$\psi^Q = -l_q i_{sg}^Q + l_{akq} i_k^Q, \tag{39}$$

$$\psi_f = l_{ff}i_f + l_{fkd}i_k^D - l_{afd}i_{sg}^D, \tag{40}$$

$$\psi_k^D = l_{fkd}i_f + l_{kkd}i_k^D - l_{akd}i_{sq}^D, \tag{41}$$

$$\psi_k^Q = l_{kkq} i_k^Q - l_{akq} i_Q. \tag{42}$$

The linearization of the previous equations yields the following state-space matrices:

$$A_{EM} = M_R M_L^{-1} + M_\omega, \qquad (43)$$

where  $M_L^{-1}$  is the inverse of  $M_L$  which is derived from (38)–(42) as follows:

$$\begin{bmatrix} \Delta \psi^D \\ \Delta \psi_f \\ \Delta \psi_k^D \\ \Delta \psi^Q \\ \Delta \psi^Q \\ \Delta \psi_k^Q \end{bmatrix} = M_L \begin{bmatrix} \Delta i_{sg}^D \\ \Delta i_f \\ \Delta i_k^D \\ \Delta i_{sg}^Q \\ \Delta i_{sg}^Q \\ \Delta i_k^Q \end{bmatrix},$$
(46)

where

$$M_L^{-1} = \begin{bmatrix} K_{id\_fd} & K_{id\_ff} & K_{id\_fkd} & 0 & 0\\ K_{if\_fd} & K_{if\_ff} & K_{if\_fkd} & 0 & 0\\ K_{ikd\_fd} & K_{ikd\_ff} & K_{ikd\_fkd} & 0 & 0\\ 0 & 0 & 0 & K_{iq\_fq} & K_{iq\_fkq}\\ 0 & 0 & 0 & K_{ikq\_fq} & K_{ikq\_fkq} \end{bmatrix} .$$

$$(47)$$

The rest of the matrices of the electromagnetic model are:

$$B_{EM1} = \begin{bmatrix} 0\\1\\0\\0\\0\\0\\0 \end{bmatrix}, B_{EM2} = \begin{bmatrix} 1&0\\0&0\\0&0\\0&1\\0&0 \end{bmatrix}, B_{EM3} = \begin{bmatrix} 0\\\Psi_q\\0\\0\\-\Psi_d\\0 \end{bmatrix}, \quad (48)$$

$$C_{EM1} = \begin{bmatrix} (K_{iq\_fq} - K_{id\_fd})\Psi_Q + K_{iq\_fkq}\Psi_k^{\omega} \\ -K_{id\_ff}\Psi_Q \\ -K_{id\_fk}\Psi_Q \\ (K_{iq\_fq} - K_{id\_fd})\Psi_D - K_{id\_fd}\Psi_f - K_{id\_fkd}\Psi_k^D \end{bmatrix}^{(49)},$$

$$C_{EM2} = \begin{bmatrix} K_{id\_fd} & K_{id\_ff} & K_{id\_fkd} & 0 & 0 \\ 0 & 0 & 0 & K_{iq\_fq} & K_{iq\_fkq} \end{bmatrix},$$
(50)

$$C_{EM3} = \begin{bmatrix} K_{if\_fd} & K_{if\_ff} & K_{if\_fkd} & 0 & 0 \end{bmatrix},$$
(51)

where parameters inside matrices are defined here:

$$K_{id\_fd} = \frac{-l_{fkd}^2 + l_{ff}l_{kkd}}{K_{id\_f}}, \ K_{id\_ff} = \frac{(l_{fkd} - l_{kkd})l_{ad}}{K_{id\_f}},$$
(52)

$$K_{id\_fkd} = \frac{(-l_{ff} + l_{fkd})l_{ad}}{K_{id\_f}}, K_{if\_fd} = \frac{(-l_{fkd} + l_{kkd})l_{ad}}{K_{id\_f}},$$
(53)

$$K_{if\_ff} = \frac{l_{ad}^2 - l_d l_{kkd}}{K_{id\_f}}, \quad K_{if\_fkd} = \frac{-l_{ad}^2 + l_d l_{fkd}}{K_{id\_f}}, \quad (54)$$

$$K_{ikd\_fd} = \frac{(l_{ff} - l_{fkd})l_{ad}}{K_{id\_f}}, \quad K_{ikd\_ff} = \frac{-l_{ad}{}^2 + l_d l_{fkd}}{K_{id\_f}}, \quad (55)$$

$$K_{ikd\_fkd} = \frac{l_{ad}^2 - l_d l_{ffd}}{K_{id\_f}},\tag{56}$$

$$K_{id}f = l_{ad}^{2} (l_{ff} + l_{kkd} - 2l_{fkd}) + l_{d} l_{fkd}^{2} - l_{d} l_{ff} l_{kkd}, \quad (57)$$

$$K_{iq_f} = l_{aq}^2 - l_{kkq}l_q, \ K_{iq_fq} = \frac{l_{kkq}}{K_{iq_f}},$$
 (58)

$$K_{iq\_fkq} = \frac{-l_{aq}}{K_{iq\_f}}, \ K_{ikq\_fq} = \frac{l_{aq}}{K_{iq\_f}}, \ K_{ikq\_fkq} = \frac{-l_q}{K_{iq\_f}}.$$
 (59)

2) *State-space matrices of the shaft:* The matrices that define the linear model of the shaft are:

$$B_{S1} = [1/2H], \quad B_{S2} = [-1/2H].$$
 (60)

*3)* State-space matrices of the excitation model: The following matrices are used in the excitation model:

$$A_E = [-(S_E + K_E)/T_E], \ B_{E1} = [1/T_E], \ C_E = [1].$$
 (61)

4) State-space matrices of the diesel engine: The model of the diesel engine is linearised around the operating point, yielding the following state-space matrices:

$$A_D = \left[\frac{-(\sigma + F)}{\tau_m(\sigma + T_m)^2}\right], \quad B_D = \left[\frac{1}{\tau_m(\sigma + T_m)}\right]. \tag{62}$$

5) State space matrices of the AVR: The matrices below represent the state-space model of the linearised AVR:

$$B_{AVR1} = [1], \ B_{AVR2} = \left[\frac{-V_{sg}^D}{\sqrt{V_{sg}^{D^2} + V_{sg}^{Q^2}}} - \frac{-V_{sg}^Q}{\sqrt{V_{sg}^{D^2} + V_{sg}^{Q^2}}}\right], \ (63)$$

$$C_{AVR} = \begin{bmatrix} k_{iAVR} \end{bmatrix}, \quad D_{AVR1} = \begin{bmatrix} k_{pAVR} \end{bmatrix}, \quad (64)$$
$$\begin{bmatrix} -V_{ex}^D k_{pAVR} & -V_{ex}^Q k_{pAVR} \end{bmatrix}$$

$$D_{AVR2} = \left[ \frac{\frac{V_{sg} m_{PAVR}}{\sqrt{V_{sg}^{D^2} + V_{sg}^{Q^2}}} - \frac{V_{sg} m_{PAVR}}{\sqrt{V_{sg}^{D^2} + V_{sg}^{Q^2}}} \right].$$
(65)

6) State space matrices of the speed governor: The matrices below represent the state-space model of the speed governor of the SG:

$$B_{Gov1} = [1], B_{Gov2} = [-1], G_{ov} = [k_{iGov}],$$
(66)  
$$D_{Gov1} = [k_{pGov}], D_{Gov2} = [-k_{pGov}].$$
(67)

7) State-Space Matrices of the diesel Gen-Set Model: The auxiliary matrices of the diesel generator aggregated small-signal model are presented below:

$$\begin{bmatrix} \Delta x_{SG} \end{bmatrix} = \begin{bmatrix} \Delta \alpha, \Delta \beta, \Delta v_f, \Delta t_m, \Delta \psi_d, \Delta \psi_f, \Delta \psi_{kd}, \Delta \psi_q, \Delta \psi_{kq}, \Delta \omega_r \end{bmatrix}^T$$
(68)
$$\begin{bmatrix} \Delta u_{SG} \end{bmatrix} = \begin{bmatrix} \Delta v_{sg}^*, \ \Delta \omega_{sg}^*, \ \Delta v_{pcc}^{DQ} \end{bmatrix}^T$$
(70)

$$B_{SG1} = \frac{1}{U_B} \left[ B_{AVR1}, \ 0, \ B_{E1} D_{AVR1}, \ 0, \ 0, \ 0 \right]^T, \tag{71}$$

$$B_{SG2} = \frac{1}{\omega_B} \begin{bmatrix} 0, \ B_{Gov1}, \ 0, \ B_D D_{Gov1}, \ 0, \ 0 \end{bmatrix}^T$$
(72)

$$B_{SG3} = \frac{1}{U_B} \left[ B_{AVR2}, 0, B_{E1} D_{AVR2}, 0, B_{EM2}, 0 \right]^T, \quad (73)$$

$$C_{SG1} = I_B \left[ 0, 0, 0, 0, C_{EM2}, 0 \right]^T, \ C_{SG2} = \omega_B \left[ 0, \vdots, 0, 1 \right]^T.$$
(74)

#### B. State Space Models of Converters Subsystems

1) LCL Filter Model: The complete small-signal model of the LCL filter can be found in the literature [19]. If the state variables are  $x_{LCL} = [\Delta i_i^{dq} \Delta v_c^{dq} \Delta i_o^{dq}]^T$ , then:

$$\begin{aligned} [\dot{x}_{LCL}]_i &= A^i_{LCL} [x_{LCL}]_i + B^i_{LCL1} \left[ \Delta v^{dq}_i \right]_i + \\ &+ B^i_{LCL2} \left[ \Delta v^{dq}_{pcc} \right]_i + B^i_{LCL3} \left[ \Delta \omega_{PLL} \right]_i. \end{aligned} \tag{75}$$

where i denotes the *i*th converter. The *LCL* filter auxiliary matrices are presented below:

$$A_{LCL}^{i} = \begin{bmatrix} \frac{-R_{i}}{-\Omega_{i}} & \Omega_{i} & \frac{-1}{L_{i}} & 0 & 0 & 0\\ -\Omega_{i} & \frac{-R_{i}}{L_{i}} & 0 & \frac{-1}{L_{i}} & 0 & 0\\ \frac{1}{C_{d}} & 0 & 0 & \Omega_{i} & \frac{-1}{C_{d}} & 0\\ 0 & \frac{1}{C_{d}} & -\Omega_{i} & 0 & 0 & \frac{-1}{C_{d}}\\ 0 & 0 & \frac{1}{L_{o}} & 0 & \frac{-R_{o}}{L_{o}} & \Omega_{i}\\ 0 & 0 & 0 & \frac{1}{L_{o}} & -\Omega_{i} & \frac{-R_{o}}{L_{o}} \end{bmatrix}_{i},$$
(76)  
$$B_{LCL1}^{i} = \begin{bmatrix} \frac{1}{L_{i}} & 0\\ 0 & \frac{1}{L_{i}}\\ 0 & 0\\ 0 & 0\\ 0 & 0 \end{bmatrix}_{i}, B_{LCL2}^{i} = \begin{bmatrix} 0 & 0\\ 0 & 0\\ 0 & 0\\ 0 & 0\\ \frac{-1}{L_{o}} & 0\\ 0 & \frac{-1}{L_{o}} \end{bmatrix}_{i},$$
(77)  
$$B_{LCL3}^{i} = \begin{bmatrix} I_{iq}, -I_{id}, V_{oq}, -V_{od}, I_{oq}, -I_{od} \end{bmatrix}_{i}^{T}.$$
(78)

2) Current controller: A current controller generates the command for the modulation stage of the VSC  $(v_i^{*dq})_i$  [19]:

$$\left[\Delta \dot{\gamma}^{dq}\right]_i = B^i_{CC1} \left[\Delta i^{*dq}_i\right]_i + B^i_{CC2} [x_{LCL}]_i, \tag{79}$$

$$\begin{bmatrix} \Delta v_i^{*dq} \end{bmatrix}_i = C_{CC}^i \begin{bmatrix} \Delta \gamma^{dq} \end{bmatrix}_i + D_{CC1}^i \begin{bmatrix} \Delta i_i^{*dq} \end{bmatrix}_i + D_{CC2}^i [x_{LCL}]_i.$$
(80)

The current controller is defined by the following space state matrices:

$$B_{CC1}^{i} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{i}, B_{CC2}^{i} \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \end{bmatrix}_{i}, \quad (81)$$

$$C_{CC}^{i} = \begin{bmatrix} k_{ic} & 0\\ 0 & k_{ic} \end{bmatrix}_{i}, \ D_{CC1}^{i} = \begin{bmatrix} k_{pc} & 0\\ 0 & k_{pc} \end{bmatrix}_{i},$$
(82)

$$D_{CC2}^{i} = \begin{bmatrix} -k_{pc} & -w_{n}L_{i} & 0 & 0 & 0 \\ w_{n}L_{i} & -k_{pc} & 0 & 0 & 0 \end{bmatrix}_{i}^{i},$$

*3) Current Set-Point Calculation:* The set-point is calculated in terms of the active and reactive power references. These equations are non-linear, so they have to be linearised. The non-linear equations are:

$$i^{*d} = \frac{p^* v_{pcc}^d + q^* v_{pcc}^q}{v_{pcc}^d + v_{pcc}^q}, \quad i^{*q} = \frac{-q^* v_{pcc}^d + p^* v_{pcc}^q}{v_{pcc}^d + v_{pcc}^q}, \quad (83)$$

while the linearised model is:

$$\left[\Delta i_i^{*dq}\right]_i = D_{I1}^i \begin{bmatrix} \Delta p^* \\ \Delta q^* \end{bmatrix}_i + D_{I2}^i \begin{bmatrix} \Delta v_{pcc}^{dq} \end{bmatrix}_i, \quad (84)$$

with

$$D_{I1}^{i} = \begin{bmatrix} \frac{V_{bd}}{V_{bd}^{2} + V_{bq}^{2}} & \frac{V_{bq}}{V_{bd}^{2} + V_{bq}^{2}} \\ \frac{V_{bq}}{V_{bd}^{2} + V_{bq}^{2}} & \frac{-V_{bd}}{V_{bd}^{2} + V_{bq}^{2}} \end{bmatrix}_{i}$$
(85)

4) State space matrices of the PLL: The state-space model of the PLL is defined by the matrices below:

$$A_{PLL}^{i} = \begin{bmatrix} A_{PLL_{ee}} & A_{PLL_{ed}} \\ A_{PLL_{de}} & A_{PLL_{dd}} \end{bmatrix}_{i} = \begin{bmatrix} 0 & k_{ipll} T_{Vd}^{-1} \\ 1 & k_{ppll} T_{Vd}^{-1} \end{bmatrix}_{i}, \quad (87)$$

$$B_{PLL1}^{i} = \begin{bmatrix} B_{PLL1e} \\ B_{PLL1d} \end{bmatrix}_{i} = \begin{bmatrix} k_{ipll} T_{Sd}^{-1} \\ k_{ppll} T_{Sd}^{-1} \end{bmatrix}_{i},$$
(88)

$$B_{PLL2}^{i} = \begin{bmatrix} B_{PLL2e} \\ B_{PLL2d} \end{bmatrix}_{i} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}_{i}.$$
(89)

5) Voltage controller: A PI controller is used to control the voltage of the ac capacitor. This controller is modelled as in [19]. The linearised equations are:

$$\left[\Delta \dot{\phi}^{dq}\right]_i = B^i_{VC1} \left[\Delta v_c^{*dq}\right]_i + B^i_{VC2} [x_{LCL}]_i, \tag{90}$$

$$\left[\Delta i_i^{*dq}\right]_i = C_{VC}^i \left[\Delta \phi^{dq}\right]_i + D_{VC1}^i \left[\Delta v_c^{*dq}\right]_i + D_{VC2}^i [x_{LCL}]_i$$
(91)

The state-space matrices of the voltage controller are:

$$B_{VC1}^{i} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{i}, B_{VC2}^{i} \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix}_{i}, \quad (92)$$

$$C_{VC}^{i} = \begin{bmatrix} k_{iv} & 0\\ 0 & k_{iv} \end{bmatrix}_{i}, \ D_{VC1}^{i} = \begin{bmatrix} k_{pv} & 0\\ 0 & k_{pv} \end{bmatrix}_{i},$$
(93)

$$D_{VC2}^{i} = \begin{bmatrix} 0 & 0 & -k_{pv} & -w_n C_f & 0 & 0\\ 0 & 0 & w_n C_f & -k_{pv} & 0 & 0 \end{bmatrix}_{i}.$$
 (94)

6) Droop controller: Frequency and voltage droop controllers are implemented together with active and reactive power low-pass filters to obtain the voltage and frequency set-points [19]. Instantaneous active and reactive powers are calculated as follows:

$$p = v_o^d i_o^d + v_o^q i_o^q, \quad q = -v_o^d i_o^q + v_o^q i_o^d.$$
(95)

In addition,  $\omega_{nd}$  and  $v_{nd}$  are considered as inputs of the system. Calling  $x_{pq} = [\Delta \delta \ \Delta \tilde{P} \ \Delta \tilde{Q}]$ , then:

$$\begin{bmatrix} \dot{x}_{pq} \end{bmatrix}_{i} = A_{P}^{i} \begin{bmatrix} x_{pq} \end{bmatrix}_{i} + B_{P1}^{i} \begin{bmatrix} x_{LCL} \end{bmatrix}_{i} + B_{PIn}^{i} \begin{bmatrix} \Delta \omega_{nd} \\ \Delta v_{nd}^{dq} \end{bmatrix}_{i} + B_{PV}^{i} \begin{bmatrix} \Delta v_{o}^{dq} \end{bmatrix}_{i}, \quad (96)$$

$$\left[\Delta\omega_{com}\right]_{i} = C_{P\omega}^{i} \left[x_{pq}\right]_{i} + D_{P\omega}^{i} \left[\frac{\Delta\omega_{nd}}{\Delta v_{nd}^{dq}}\right]_{i}, \qquad (97)$$

$$D_{I2}^{i} = \begin{bmatrix} \frac{P(V_{bd}^{2}+V_{bd}^{2})-2V_{bd}(PV_{bd}+QV_{bq})}{(V_{cd}^{2}+V_{bd}^{2})^{2}} & \frac{Q(V_{bd}^{2}+V_{bd}^{2})-2V_{bd}(PV_{bd}+QV_{bq})}{(V_{cd}^{2}+V_{bd}^{2})^{2}} \\ \frac{-Q(V_{bd}^{2}+V_{bd}^{2})-2V_{bd}(-QV_{bd}+PV_{bq})}{(V_{bd}^{2}+V_{bd}^{2})^{2}} & \frac{P(V_{bd}^{2}+V_{bd}^{2})^{2}}{(V_{bd}^{2}+V_{bd}^{2})^{2}} \end{bmatrix}_{i} \\ A^{GFe} = \begin{bmatrix} A_{PLLee} & A_{PLLed} & 0 & 0 \\ A_{PLLde} & A_{PLLdd} & 0 & 0 \\ 0 & B_{CC1}D_{I2}T_{Vd}^{-1} & 0 & B_{CC2} \\ B_{LC13}A_{PLLde} & +(B_{LC1}D_{CC1}D_{I2}+ B_{LCL1}C_{CC} & A_{LCL} + B_{LCL1}D_{CC2} \end{bmatrix} \\ A^{GFe} = \begin{bmatrix} A_{P} + B_{PV} \begin{bmatrix} T_{V}^{-1} & 0 \end{bmatrix} & 0 & 0 & B_{P1} \\ B_{VC1}C_{PV} + B_{VC3} \begin{bmatrix} T_{V}^{-1} & 0 \end{bmatrix} & A_{VC} & 0 & B_{VC2} \\ B_{LC13}A_{PLLde} & +(B_{LC1}D_{CC1}D_{I2}+ B_{LC1}C_{CC} & A_{LCL} + B_{LC1}D_{CC2} \end{bmatrix} \\ A^{GFo} = \begin{bmatrix} A_{P} + B_{PV} \begin{bmatrix} T_{V}^{-1} & 0 \end{bmatrix} & 0 & 0 & B_{P1} \\ B_{VC1}C_{PV} + B_{VC3} \begin{bmatrix} T_{V}^{-1} & 0 \end{bmatrix} & A_{VC} & 0 & B_{VC2} \\ B_{LC1}D_{CC1}D_{VC3} + B_{CC1}D_{VC3} \begin{bmatrix} T_{V}^{-1} & 0 \end{bmatrix} & B_{CC1}C_{VC} & A_{CC} & B_{CC1}D_{VC2} + B_{CC2} \\ B_{LC1}D_{CC1}D_{VC3} + B_{LC2} \end{bmatrix} \begin{bmatrix} T_{V}^{-1} & 0 \end{bmatrix} & B_{LC1}D_{CC1}C_{VC} & B_{LC1}D_{CC1}D_{VC2} + B_{CC2} \\ B_{LC1}D_{CC1}D_{VC3} + B_{LC2} \end{bmatrix} \begin{bmatrix} T_{V}^{-1} & 0 \end{bmatrix} & B_{LC1}D_{CC1}C_{VC} & B_{LC1}D_{CC1}D_{VC2} + B_{CC2} \end{bmatrix}$$
(114)

$$\left[\Delta v_c^{*dq}\right]_i = C_{PV}^i \left[x_{pq}\right]_i + D_{PV}^i \left[\frac{\Delta \omega_{nd}}{\Delta v_{nd}^{dq}}\right]_i, \qquad (98)$$

where

$$A_P^i = \begin{bmatrix} 0 & -m_P & 0\\ 0 & -\omega_c & 0\\ 0 & 0 & -\omega_c \end{bmatrix}_i,$$
(99)

$$B_{P1}^{i} = \begin{bmatrix} 0 & 0 & 0 & 0 & \omega_{c}V_{o_{d}} & \omega_{c}V_{o_{q}} \\ 0 & 0 & 0 & 0 & \omega_{c}V_{o_{q}} & -\omega_{c}V_{o_{d}} \end{bmatrix}_{i}, B_{P\omega}^{i} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}_{i},$$
(100)  
$$B_{PIn}^{i} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{i}, B_{PV}^{i} = \begin{bmatrix} 0 & 0 \\ \omega_{c}I_{o_{d}} & \omega_{c}I_{o_{q}} \\ -\omega_{c}I_{o_{q}} & \omega_{c}I_{o_{d}} \end{bmatrix}_{i},$$
(101)

$$C_{P\omega}^{i} = \begin{bmatrix} 0 & -m_{p} & 0 \end{bmatrix}_{i}, \ C_{PV}^{i} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -n_{q} \end{bmatrix}_{i},$$
(102)

$$D_{P\omega}^{i} = \begin{bmatrix} 1 \end{bmatrix}_{i}, \ D_{PV}^{i} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}_{i}.$$
 (103)

7) State-Space Matrices for Reference Frame Transformations: The following matrices are used for transformations between reference frames:

$$\left[\Delta i_{o_{DQ}}\right]_{i} = T_{S}^{i} \left[i_{o_{dq}}\right]_{i} + T_{C}^{i} \left[\Delta\delta\right]_{i}, \qquad (104)$$

$$T_{S}^{i} = \begin{bmatrix} \cos \delta_{0} & -\sin \delta_{0} \\ \sin \delta_{0} & \cos \delta_{0} \end{bmatrix}_{i},$$
(105)

$$T_C^i = \begin{bmatrix} -I_{od} \sin \delta_0 - I_{oq} \cos \delta_0 \\ I_{od} \cos \delta_0 - I_{oq} \sin \delta_0 \end{bmatrix}_i,$$
(106)

$$\left[\Delta v_{b_{dq}}\right]_{i} = T_{S}^{-1i} \left[v_{b_{DQ}}\right]_{i} + T_{V}^{-1i} \left[\Delta\delta\right]_{i}.$$
 (107)

$$T_V^{-1i} = \begin{bmatrix} -V_{bd}\sin\delta_0 + V_{bq}\cos\delta_0\\ -V_{bd}\cos\delta_0 - V_{bq}\sin\delta_0 \end{bmatrix}_i,$$
(108)

where  $\delta_0$  has been used instead of  $\Delta$  to avoid confusion with the incremental operator " $\Delta$ ".

# C. State-Space Matrices of the GFe-VSC

The matrices used in the GFe-VSC model are:

$$B_{PQ}^{GFe} = \begin{bmatrix} 0, \ 0, \ B_{CC1}D_{I1}, \ B_{LCL1}D_{CC1}D_{I1} \end{bmatrix}^T, \quad (110)$$

$$B_{V}^{GFe} = \begin{bmatrix} B_{PLL1e} \\ B_{PLL1d} \\ B_{CC1}D_{I2}T_{S}^{-1} \\ (B_{LCL1}D_{CC1}D_{I2} + B_{LCL2})T_{S}^{-1} \\ + B_{LCL3}B_{PLL1d} \end{bmatrix},$$
(111)

$$B_{\omega}^{GFe} = \begin{bmatrix} B_{PLL2e}, B_{PLL2d}, 0, 0 \end{bmatrix}^{T}, \qquad (112)$$

$$C^{GFe} = \begin{bmatrix} 0 & T_C & 0 & T_S \end{bmatrix}.$$
(113)

*D. State-Space Matrices of the Droop-Controlled Converter* The matrices of the GFo-VSC are:

$$B_{In}^{GFo} = \begin{bmatrix} B_{PIn} \\ B_{VC1} \begin{bmatrix} 0 & D_{PV} \end{bmatrix} \\ B_{CC1} D_{VC1} \begin{bmatrix} 0 & D_{PV} \end{bmatrix} \\ \begin{bmatrix} B_{LCL3} D_{P\omega} & B_{LCL1} D_{CC1} D_{VC1} D_{PV} \end{bmatrix} \end{bmatrix}, \quad (115)$$

$$B_{V}^{GFo} = \begin{bmatrix} B_{PV}T_{s}^{-1} \\ B_{VC3}T_{s}^{-1} \\ B_{CC1}D_{VC3}T_{s}^{-1} \\ (B_{LCL2} + B_{LCL1}D_{CC1}D_{VC3})T_{s}^{-1} \end{bmatrix}, \quad (116)$$

$$B_{\omega com}^{GFo} = \begin{bmatrix} B_{P\omega} \\ [0] \end{bmatrix}, \quad C_I^{GFo} = \begin{bmatrix} [T_C \quad 0] & [0] \quad [0 \quad T_C] \end{bmatrix}, \quad (117)$$
$$C_{\omega}^{GFo} = \begin{bmatrix} C_{P\omega} \quad 0 \end{bmatrix}, \quad D_{\omega}^{GFo} = \begin{bmatrix} D_{P\omega} \quad 0 \end{bmatrix}. \quad (118)$$

# E. State-Space Matrices of Linear R - L Load

The state-space matrices of the load are:

$$A_L = \begin{bmatrix} \frac{-R_l}{L_l} & \Omega_{sg} \\ -\Omega_{sg} & \frac{-R_l}{L_l} \end{bmatrix}, \ B_{L1} = \begin{bmatrix} \frac{1}{L_l} & 0 \\ 0 & \frac{1}{L_l} \end{bmatrix},$$
(119)

$$B_{L2} = \begin{bmatrix} I_L^Q \\ -I_L^D \end{bmatrix}, \ C_L = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$
(120)

# F. State-Space Matrices of the Auxiliary Resistor

The state-space matrices of the auxiliary R are:

$$D_{R1} = \begin{bmatrix} \frac{1}{R_x} & 0\\ 0 & \frac{1}{R_x} \end{bmatrix}, \ D_{R2} = \begin{bmatrix} \frac{-1}{R_x} & 0\\ 0 & \frac{-1}{R_x} \end{bmatrix}.$$
(121)



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